LIMEIRA 2019

51º SBPO

# Problema de Corte: modelagem matemática e resolução com auxílio de solver

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# Conteúdo

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- Problema da mochila;
- Formulação matemática de Kantorovich;
- Introdução CPLEX

## Parte II

- Formulação matemática de  Gilmore e Gomory.
- Formulação matemática de Valério de Carvalho.
- Implementações no CPLEX.

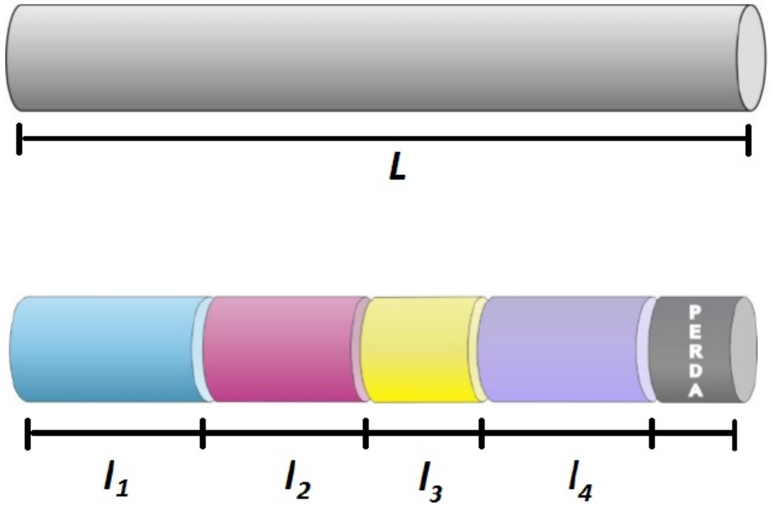
# O problema de corte de estoque

## Definição:

Dados objetos grandes, geralmente disponíveis em estoque, o PCE – Problema de Corte de Estoque consiste em determinar como cortá-los em peças menores, chamadas de itens, de acordo com algum critério de otimização.

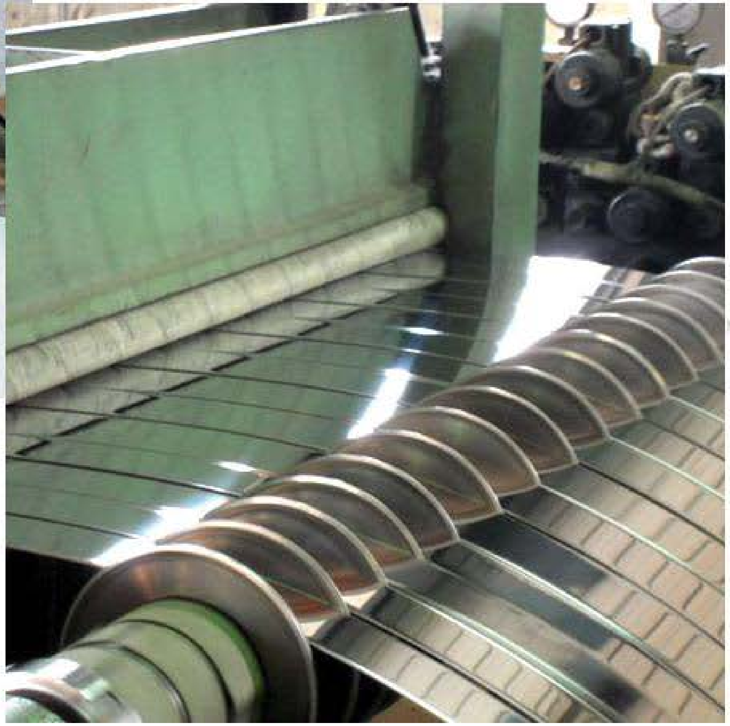
# O problema de corte de estoque

Unidimensional



# O problema de corte de estoque

## Unidimensional

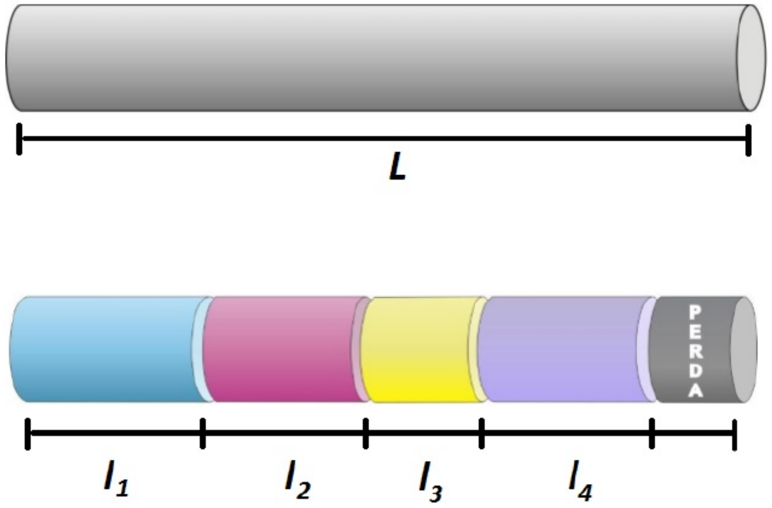


# O problema de corte de estoque

Unidimensional

# O problema de corte de estoque

## Unidimensional



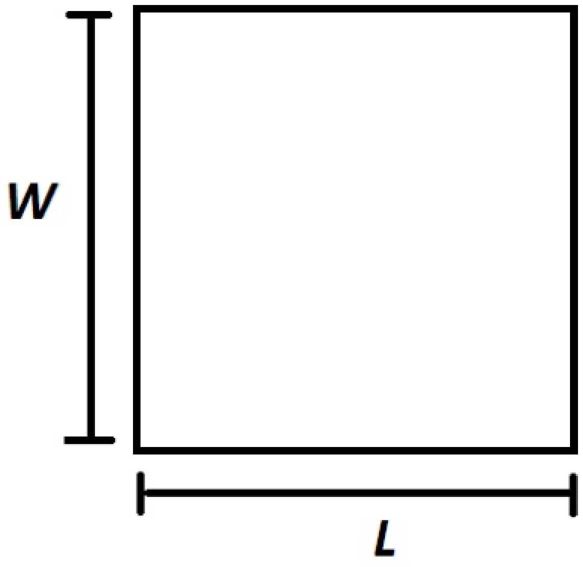

Indústria de bobinas de aço, bobinas (jumbo) de papel, esquadrias de madeira, tubos para fabricação de aeronaves entre outros.

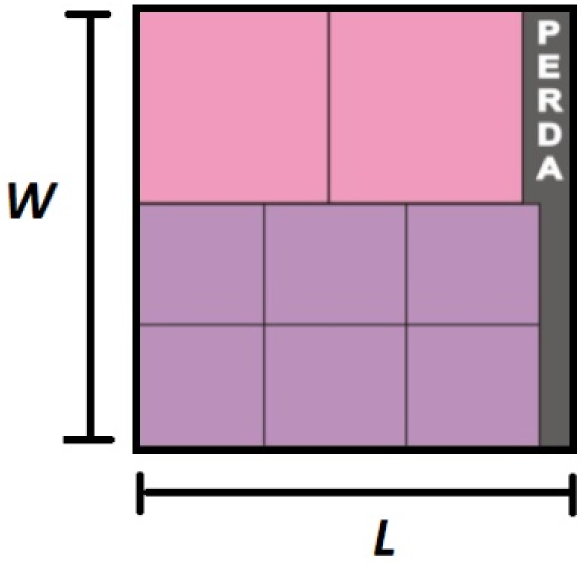
Busca no Google Scholar:

- Gau e Wäscher e Gau (1995);
- Vance (1998);
- Belov e Scheithauer (2002);
- Stadtler (1990);
- Umetani et al (2003);
- Vanderbeck (2000);
- Poldi e Arenales (2009);
- Gradisar e Trkman (2005);
- Cherri et al (2014);
- Vahrenkamp (1996);
- Cui e Yang (2010);
- Scheithauer e Terno (1997);
- Leao (2013);
- .....

# O problema de corte de estoque

Bidimensional



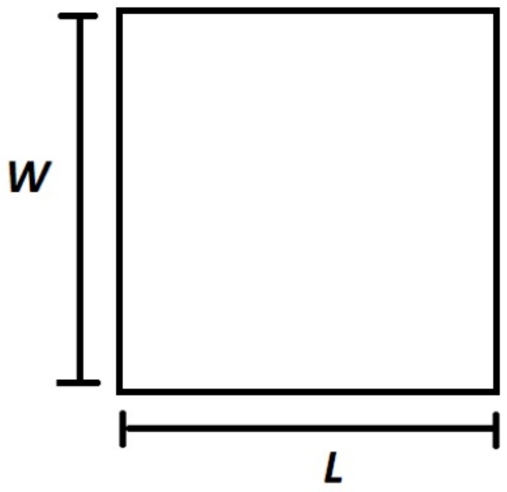
# O problema de corte de estoque

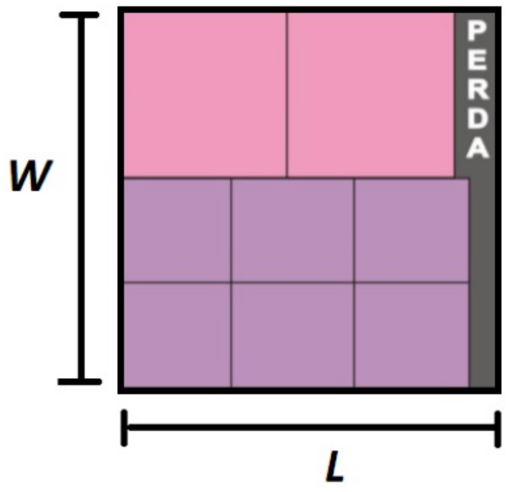


Bidimensional

# O problema de corte de estoque

## Bidimensional



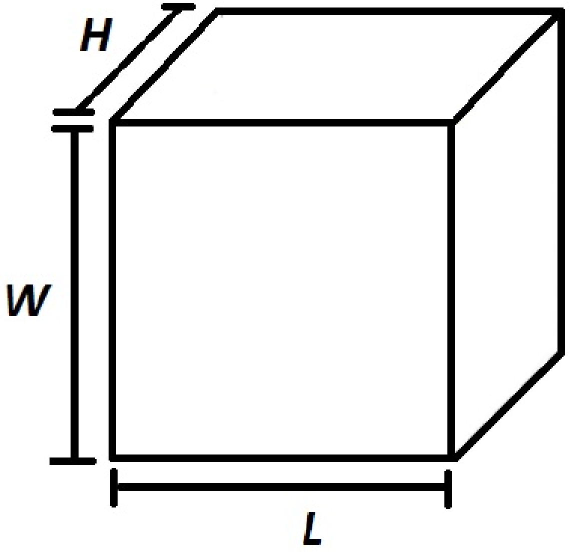



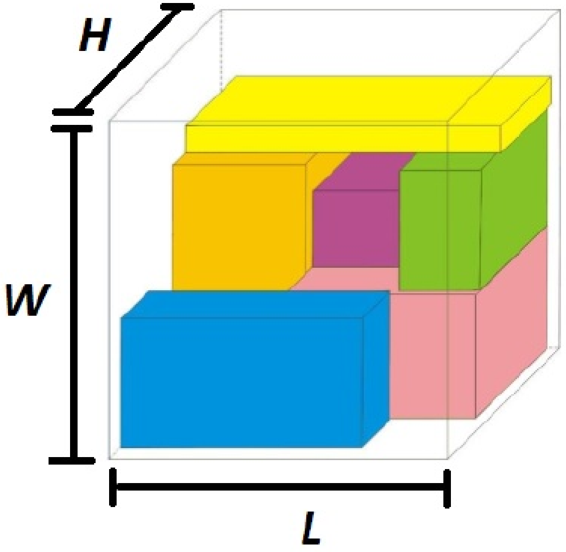

Indústria de móveis, vidros, papeis, etc

Busca no Google Scholar:

- Wang (1983);
- Kenyon e Rémila (2000);
- Gilmore e Gomory (1965);
- Steudel (1979);
- Art Jr (1966);
- Christofides e Whitlock (1977);
- Vanderbeck (2001);
- Beasley (1985);
- Herz (1972);
- Israno e Sanders (1982);
- Morabito (1992);
- Ghidini (2009);
- .....

Tridimensional



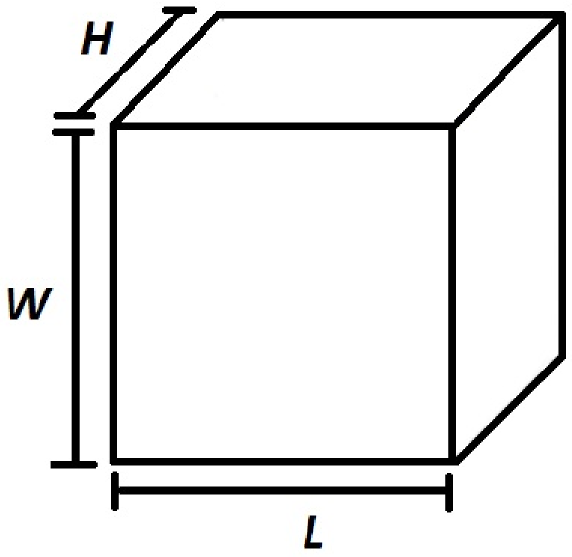
# O problema de corte de estoque

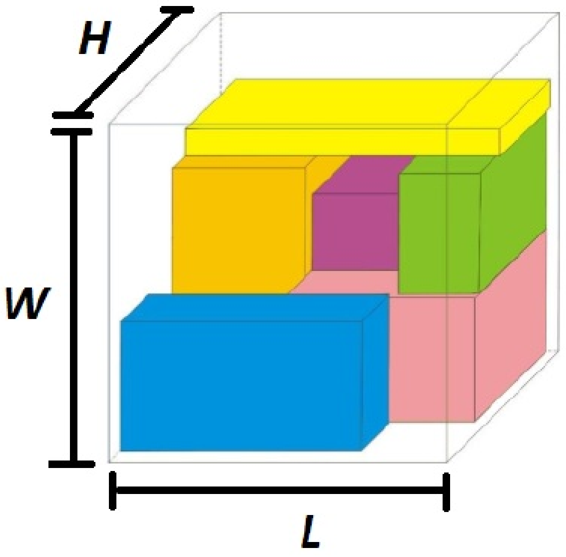
Tridimensional

Tridimensional

# O problema de corte de estoque

## Tridimensional




Busca no Google Scholar:

- Morabito e Arenales (1992);
- Antonio, Chu et al (1996);
- Moreira, Oliveira, Gomes (2007);
- Queiroz, Miyazawa e Wakabayashi;
- Hokama, Miyazawa e Xavier (2016);

- Bordfeldt e Wäscher (2013) ***review***
- .....

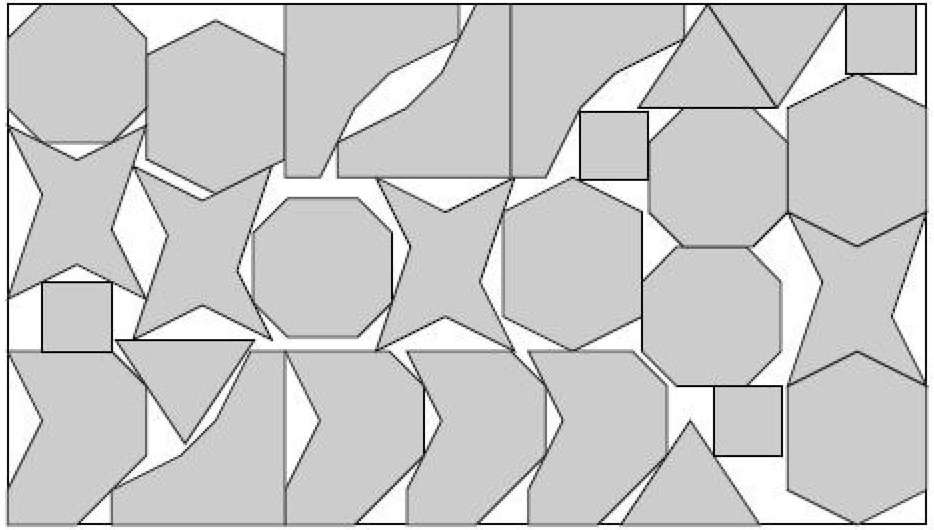
Corte de espuma, isopor, etc.

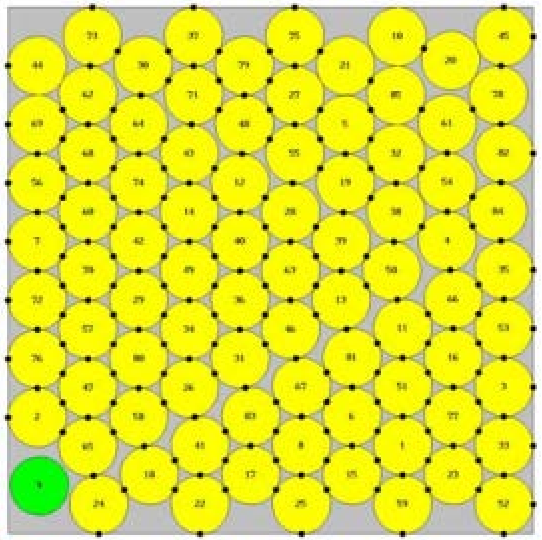
Carregamento de contêineres ou paletes.

# O problema de corte de estoque



Irregular (*nesting problems*)





# O problema de corte de estoque

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- Único/vários objetos;
- Minimização do número de padrões de corte
- Aproveitamento de sobras
- Data de entrega
- Integrações
    - PCE + problema de dimensionamento de lotes;
    - PCE + sequenciamento da produção;
    - PCE + roteamento;
    - PCE + PDL + sequenciamento;
    - etc....
- Etc.....

# O problema de corte de estoque

## Tipologia Dyckhoff (1990)

| | |
|---|---|
| 1. | Dimensão |
| | (1) unidimensional |
| | (2) bidimensional |
| | (3) tridimensional |
| | ($N$) N-dimensional com $N > 3$ |

| | |
|---|---|
| 2. | Tipo de atribuição |
| | (B) usar todos os objetos e uma seleção dos itens |
| | (V) usar uma seleção de objetos e todos os itens |

| | |
|---|---|
| 3. | Classificação dos objetos |
| | (O) um objeto |
| | (I) vários objetos de um mesmo tipo |
| | (D) vários objetos de tipos diferentes |

| | |
|---|---|
| 4. | Classificação dos itens |
| | (F) alguns itens de tipos diferentes |
| | (M) muitos itens e muitos tipos |
| | (R) muitos itens e alguns tipos (não congruentes) |
| | (C) congruentes |

European Journal of Operational Research 44 (1990) 145–159
North-Holland
145

# A typology of cutting and packing problems

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*RWTH Aachen, Templergraben 64, D-5100 Aachen, Federal Republic of Germany*

**Abstract:** Cutting and packing problems appear under various names in literature, e.g. cutting stock or trim loss problem, bin or strip packing problem, vehicle, pallet or container loading problem, nesting problem, knapsack problem etc. The paper develops a consistent and systematic approach for a comprehensive typology integrating the various kinds of problems. The typology is founded on the basic logical structure of cutting and packing problems. The purpose is to unify the different use of notions in the literature and to concentrate further research on special types of problems.

**Keywords:** Cutting, packing, production, distribution, engineering

## 1. Introduction

The topic of cutting and packing (abbreviated by C&P in the following) is characterized by the fact that problems of essentially the same logical

Table 1 also reflects the rapid development as well as the wide dispersion of research on this topic. With very few exceptions (Kantorovich, 1939; Brooks et al., 1940) scientific work started about thirty five years ago. Since then, there has

# O problema de corte de estoque

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## Tipologia Wäscher et al (2007)

Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
European Journal of Operational Research 183 (2007) 1109–1130
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

### An improved typology of cutting and packing problems

Gerhard Wäscher *, Heike Haußner, Holger Schumann

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Received 30 September 2004; accepted 15 December 2005
Available online 19 June 2006

**Abstract**

The number of publications in the area of Cutting and Packing (C&P) has increased considerably over the last two decades. The typology of C&P problems introduced by Dyckhoff [Dyckhoff, H., 1990. A typology of cutting and packing problems. European Journal of Operational Research 44, 145–159] initially provided an excellent instrument for the organisation and categorisation of existing and new literature. However, over the years also some deficiencies of this typology

# O problema de corte de estoque

**Review** Delorme, Iori e Martello (2016)

European Journal of Operational Research 255 (2016) 1–20

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Invited Review

## Bin packing and cutting stock problems: Mathematical models and exact algorithms

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A R T I C L E I N F O

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Received 26 March 2015
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Available online 22 April 2016

*Keywords:*
Bin packing
Cutting stock
Exact algorithms
Computational evaluation

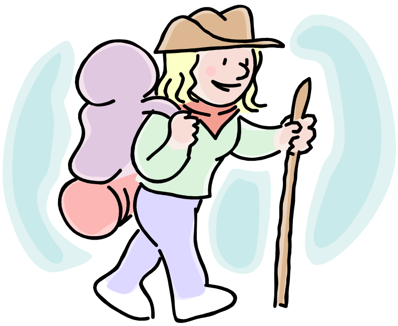
A B S T R A C T

We review the most important mathematical models and algorithms developed for the exact solution of the one-dimensional bin packing and cutting stock problems, and experimentally evaluate, on state-of-the art computers, the performance of the main available software tools.

# Problema da mochila (*knapsack problem*)



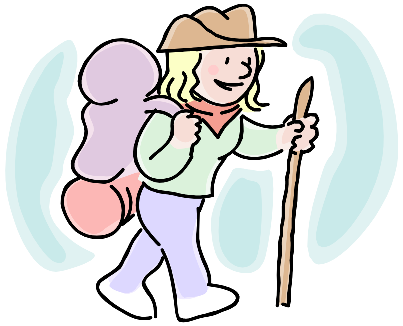
Suponha que um alpinista deva carregar sua mochila selecionando alguns itens, dentre vários disponíveis, para carregar em sua mochila em uma expedição, sempre levando em conta a capacidade da mochila.



A cada item é atribuído um valor de utilidade e o alpinista deve selecioná-los buscando maximizar o valor de utilidade total.

# Problema da mochila (*knapsack problem*)

Considere:



$m$ :  número de tipos de itens;

$v_i$  : valor de utilidade do item tipo i;

$p_i$ : peso do item tipo i;

$L$ : capacidade de mochila.

# Problema da mochila (*knapsack problem*) binário

Variável de decisão:

$$x_i = \begin{cases} 1 & \textit{se o item i é colocado na mochila} \\ 0 & \textit{caso contrário} \end{cases} :$$

## Formulação matemática:

$$\max \sum_{i=1}^{m} v_i \, x_i$$

Maximiza valor de utilidade

$$\text{sujeito a}: \begin{cases} \displaystyle\sum_{i=1}^{m} p_i x_i \leq L \\ x_i \in \mathbb{B}, i = 1, \dots, m \end{cases}$$

Respeita a capacidade da mochila

Domínio das variáveis

# Problema da mochila (*knapsack problem*) binário

## Exemplo:

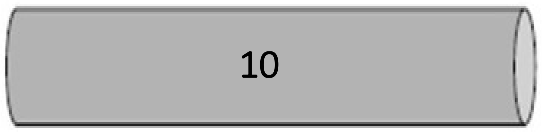
$L = 10$ : capacidade de mochila;

$m = 5$ itens;

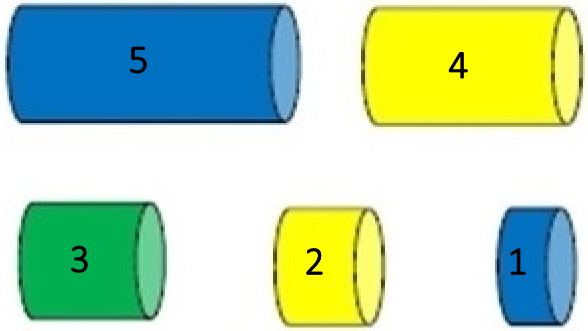
$\boldsymbol{v} = (4 \quad 6 \quad 5 \quad 3 \quad 1)^t$ ;

$\boldsymbol{p} = (5 \quad 4 \quad 3 \quad 2 \quad 1)^t$ .

Objeto:




Itens:

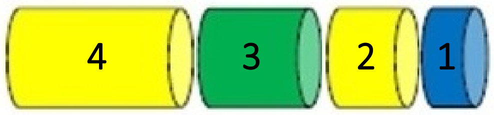



## Formulação matemática:

$$\max \quad z = 4x_1 + 6x_2 + 5x_3 + 3x_4 + x_5$$

$$\text{sujeito a :} \begin{cases} 5x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 \leq \quad 10 \\ \qquad x_i \in \mathbb{B}, i = 1, \ldots, 5 \end{cases}$$

## Solução:




$\boldsymbol{x}^* = (0 \quad 1 \quad 1 \quad 1 \quad 1)^t$

$z^* = 15$

# Problema da mochila (*knapsack problem*) inteiro

Variável de decisão:

$x_i \geq 0, i = 1, \ldots, m$ : número de itens do tipo i alocados na mochila

## Formulação matemática:

$$\max \sum_{i=1}^{m} v_i \, x_i$$

Maximiza valor de utilidade

$$\text{sujeito a}: \begin{cases} \sum_{i=1}^{m} p_i x_i \leq L \\ x_i \in \mathbb{Z}^+, i = 1, \ldots, m \end{cases}$$

Respeita a capacidade da mochila

Pode-se levar várias unidades do mesmo tipo de item na mochila

# Problema da mochila (*knapsack problem*) inteiro



Exemplo:

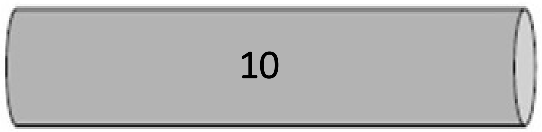
$L = 10$ : capacidade de mochila;

$m = 5$ itens;

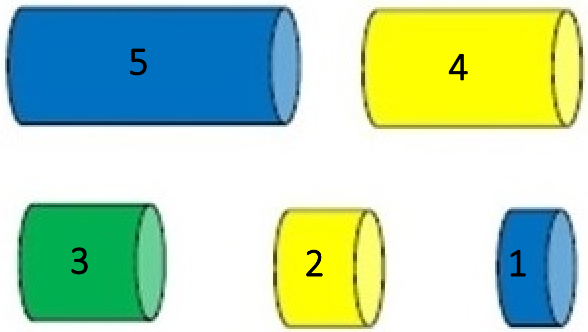
$\boldsymbol{v} = (4 \quad 6 \quad 5 \quad 3 \quad 1)^t$ ;

$\boldsymbol{p} = (5 \quad 4 \quad 3 \quad 2 \quad 1)^t$ .

Objeto:




Itens:




## Formulação matemática:

$$\max \;\; z = 4x_1 + 6x_2 + 5x_3 + 3x_4 + x_5$$

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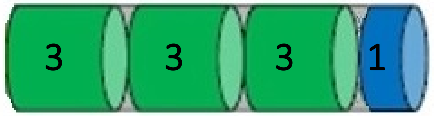
## Solução:




$\boldsymbol{x}^* = (0 \quad 0 \quad 3 \quad 0 \quad 1)^t$

$z^* = 16$

# Problema *bin-packing*



Associa-se cada item a uma mochila (*bin*), tal que o peso total dos itens em cada caixa não exceda $L$ e o número de *bins* utilizados seja mínimo

## Variáveis de decisão:

$$x_{ij} = \begin{cases} 1 & se\ o\ item\ i\ é\ colocado\ na\ mochila\ j \\ 0 & caso\ contrário \end{cases}$$

$$y_j = \begin{cases} 1 & se\ a\ mochila\ (bin)\ j\ é\ usado \\ 0 & caso\ contrário \end{cases}$$

## Formulação matemática:

$$\min \sum_{i=1}^{m} y_j$$

minimiza o total de *bins* utilizados

$$\text{sujeito a :} \begin{cases} \sum_{i=1}^{m} p_i x_{ij} \leq L_j y_j, & j = 1, \dots, m \\ \sum_{j=1}^{m} x_{ij} = 1, & i = 1, \dots, m \\ x_{ij} \in \mathbb{B},\ y_j \in \mathbb{B}, & \forall\ i, j \end{cases}$$

respeita a capacidade do *bin*, **se** for usado!

Todo item deve ser alocado em algum *bin j*

Note que o limitante superior do número de mochilas é igual ao número de itens $m$.

# Formulação de Kantorovich (1939)



L. V. Kantorovich.
*Mathematical Methods of production, planning and organization.*
Leningrad University, Leningrado, 1939, 68 pp. (em Russo)

Kantorovich foi laureado com o Prêmio Nobel em Economia, em 1975, juntamente com TjallingC. Koopmans, por suas contribuições para a teoria de alocação  ótima de recursos.

# Formulação de Kantorovich (1939)

Objetos disponíveis para corte de comprimento $L \geq 0$
Produzir itens menores de comprimentos $l_1, l_2, \dots, l_m$
e demandas $d_1, d_2, \dots, d_m$.

## Variáveis de decisão:

$$y_j = \begin{cases} 1 & se\ o\ bin\ (mochila)\ j\ é\ usado \\ 0 & caso\ contrário \end{cases}$$

$x_{ij} \geq 0$ : número de vezes que o item *i* é cortado do objeto *j*.

## Formulação matemática:

$$\min \sum_{j=1}^{m} y_j$$

$$\text{sujeito a :} \begin{cases} \sum_{i=1}^{m} l_i x_{ij} \leq L_j y_j, & j = 1, \dots, n \\ \sum_{j=1}^{n} x_{ij} \geq d_i, & i = 1, \dots, m \\ x_{ij} \in \mathbb{Z}^+, y_j \in \mathbb{B}, & \forall\, i, j \end{cases}$$

minimiza o total de objetos cortados

Comprimento total utilizado é limitado ao comprimento do objeto, **se** utilizado!

atendimento da demanda

# Formulação de Gilmore e Gomory (1961, 1963)

*Operations Research*, Vol. 9, No. 6. (Nov. - Dec., 1961), pp. 849-859.

## A LINEAR PROGRAMMING APPROACH TO THE CUTTING-STOCK PROBLEM

P. C. Gilmore and R. E. Gomory

*International Business Machines Corporation, Research Center, Yorktown, New York*

(Received May 8, 1961)

The cutting-stock problem is the problem of filling an order at minimum cost for specified numbers of lengths of material to be cut from given stock lengths of given cost. When expressed as an integer programming problem the large number of variables involved generally makes computation infeasible. This same difficulty persists when only an approximate solution is being sought by linear programming. In this paper, a technique is described for overcoming the difficulty in the linear programming formulation of the problem. The technique enables one to compute always with a matrix which has no more columns than it has rows.

SOME linear programming problems arising from combinatorial problems become intractable because of the large number of variables in

*Operations Research*, Vol. 11, No. 6. (Nov. - Dec., 1963), pp. 863-888.

## A LINEAR PROGRAMMING APPROACH TO THE CUTTING STOCK PROBLEM—PART II*

P. C. Gilmore and R. E. Gomory

*IBM Corporation, Yorktown Heights, New York*

(Received May 27, 1963)

In this paper, the methods for stock cutting outlined in an earlier paper in this JOURNAL [*Opns. Res.* 9, 849-859 (1961)] are extended and adapted to the specific full-scale paper trim problem. The paper describes a new and faster knapsack method, experiments, and formulation changes. The experiments include ones used to evaluate speed-up devices and to explore a connection with integer programming. Other experiments give waste as a function of stock length, examine the effect of multiple stock lengths on waste, and the effect of a cutting knife limitation. The formulation changes discussed are: (i) limitation on the number of cutting knives available, (ii) balancing of multiple machine usage when orders are being filled from more than one machine, and (iii) introduction of a rational objective function when customers' orders are not for fixed amounts, but rather for a range of amounts. The methods developed are also applicable to a variety of cutting problems outside of the paper industry.

IN AN earlier paper[1] we outlined a column generating procedure to overcome one of the basic difficulties associated with the cutting stock problem, the problem of too many cutting patterns, or, in linear program-

# Formulação de Gilmore e Gomory (1961, 1963)

51º SBPO LIMEIRA 2019

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* This research was supported in part by the Office of Naval Research under Contract No. Nonr 3775(00), NR 047040.

† References to earlier work can be found in reference 1. The referee has kindly brought to our attention the even earlier work of L. V. KANTOROVICH, "Mathematical Methods of Organizing and Planning Production," reprinted in *Management Sci.* 6, 366–422 (1962).

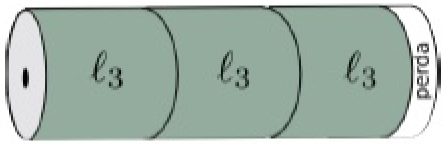
# Formulação de Gilmore e Gomory (1961, 1963)

Definição: *(padrão de corte)*

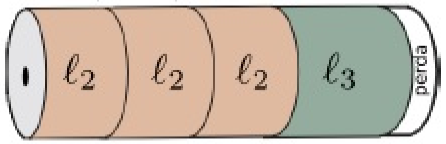
Seja $\boldsymbol{a}$ vetor (de dimensão $m$)

$$\boldsymbol{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_m \end{bmatrix}$$

tal que $a_i$ representa o número de vezes que o item tipo $i$ está no padrão de corte.




$$\boldsymbol{a} = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$




$$\boldsymbol{a}_2 = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

# Formulação de Gilmore e Gomory (1961, 1963)

Seja $n$ o número total de possíveis padrões de corte.

Assim, o j-ésimo padrão de corte é dado por:

$$\boldsymbol{a_j} = \begin{bmatrix} a_{1j} \\ \vdots \\ a_{mj} \end{bmatrix}$$

em que $a_{ij}$ representa o número de vezes que o item tipo $i$ está no padrão de corte $j$.

Seja $\boldsymbol{d}$ o vetor das demandas dos itens.

# Formulação de Gilmore e Gomory (1961, 1963)

Variável de decisão:

$\boldsymbol{x_j}$: número de objetos cortados conforme o padrão de corte $\boldsymbol{j}$.

## Formulação matemática:

$$\min \sum_{j=1}^{n} x_j$$

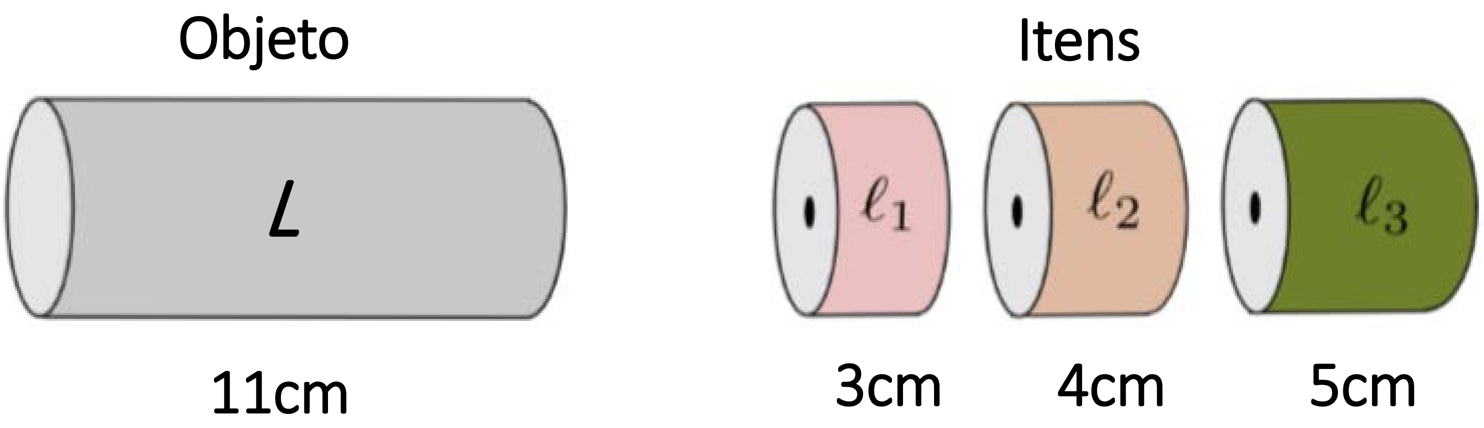
Minimiza o total de objetos cortados

$$\text{sujeito a}: \begin{cases} \displaystyle\sum_{j=1}^{n} \boldsymbol{a}_j x_j \geq \boldsymbol{d} \\ x_j \in \mathbb{Z}^+, \quad \forall\ j \end{cases}$$

atendimento da demanda

## Exemplo:

Considere o problema de corte unidimensional com um único tipo de objeto em estoque de comprimento $L$ = 11 a serem cortados para produzir três tipo de itens de comprimentos $\ell_1$ = 3cm, $\ell_2$ = 4cm e $\ell_3$ = 5cm e cujas demandas são 4, 3 e 2, respectivamente.




Demanda

$d_1 = 4$
$d_2 = 3$
$d_3 = 2$

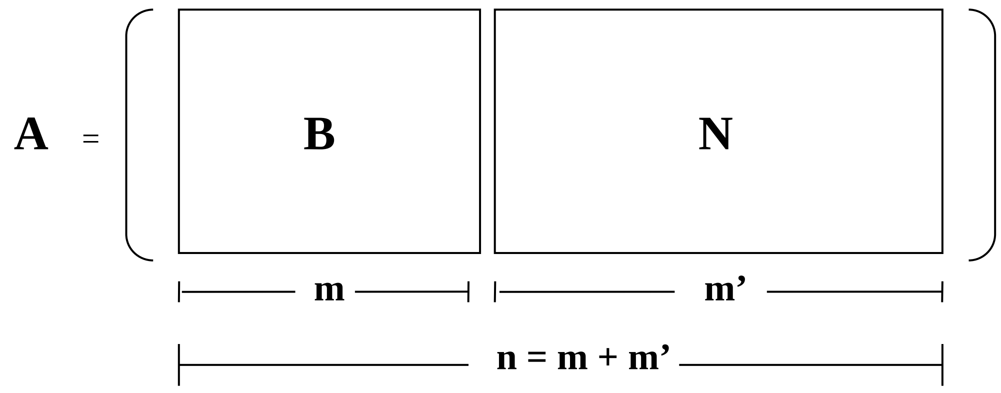
## Método de Solução

# Formulação de Gilmore e Gomory (1961, 1963)

## Método Simplex* com Geração de Colunas (GG, 1961)

* Proposto por Dantzig (1945)

1. Partição básica




2. Solução básica inicial: $\boldsymbol{B}x_B = \boldsymbol{d}$

3. Variáveis duais (vetor multiplicador simplex): $\boldsymbol{B}^t\pi = \boldsymbol{c_B}$

4. Custos relativos: $\widehat{c_{N_q}} = c_{N_q} - \pi^t a_{N_q}, \quad \forall q \in \boldsymbol{N}$

*Dantzig*: $\widehat{c_k} = \min\left\{\widehat{c_{N_q}}, \forall q\right\}$

$c_k - \pi^t a_k \geq 0 \;\Rightarrow$ solução ótima

$c_k - \pi^t a_k < 0 \;\Rightarrow$ variável $k$ entra na base

$c_k = 1, \forall\, k$

$$\min\{c_k - \pi^t a_k\} = \min\{1 - \pi^t a_k\}$$
$$= 1 - \max\{\pi^t a_k\}$$

Problema da mochila

5. Direção simplex

6. Tamanho do passo

7. Atualização e retorna ao passo 3

# Formulação de Gilmore e Gomory (1961, 1963)

Problema Mestre

$$\min \sum_{j=1}^{n} x_j$$

$$\text{sujeito a :} \begin{cases} \sum_{j=1}^{n} \boldsymbol{a}_j x_j \geq \boldsymbol{d} \\ x_j \in \mathbb{R}^+, \quad \forall\ j \end{cases}$$

$\xrightarrow{\boldsymbol{\pi}}$

$\xleftarrow{\boldsymbol{a}}$

Subproblema

$$\mathrm{g} = 1 - \max \sum_{i=1}^{m} \pi_i a_i$$

$$\text{sujeito a :} \begin{cases} \sum_{i=1}^{m} \boldsymbol{l}_i a_i \leq L \\ a_i \in \mathbb{Z}^+, \quad i =, 1, \dots, m \end{cases}$$

# Formulação de Valério de Carvalho (1999, 2001)

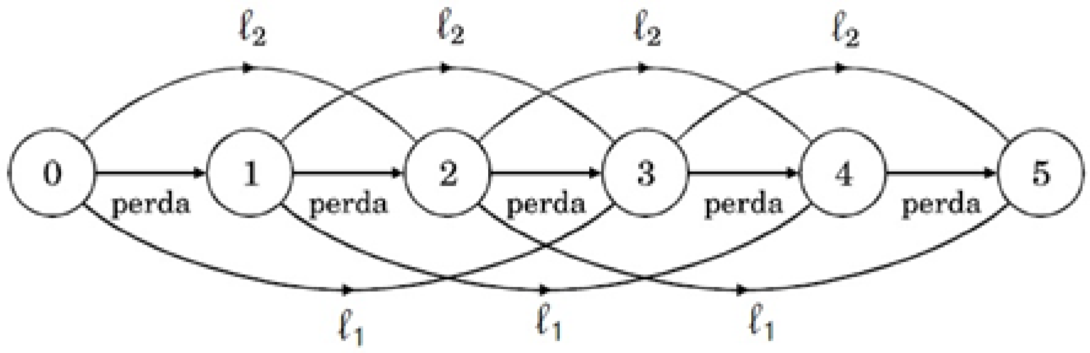
# Formulação de Valério de Carvalho (1999, 2001)

## Exemplo:

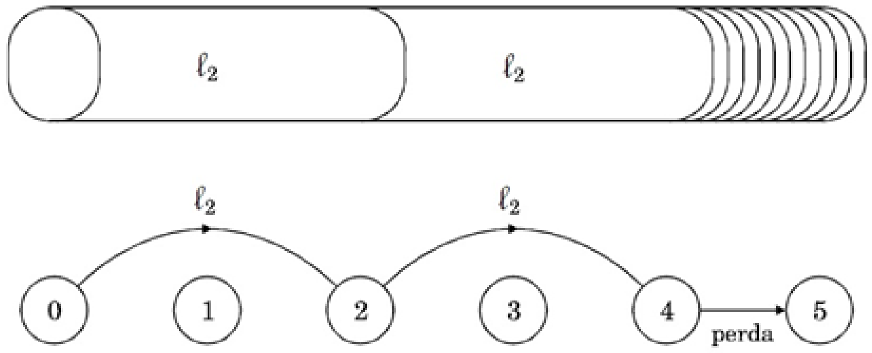
Dados:

- $L = 5$;
- $\boldsymbol{l} = (3, 2)$;

Grafo associado:




Representação de um padrão de corte:



# Formulação de Valério de Carvalho (1999, 2001)

*Minimizar* $f$ (VC1)

*Sujeito a:*

$$-\sum_{(i,j)\in A} z_{ij} + \sum_{(j,k)\in A} z_{jk} = \begin{cases} -f, & \text{se } j = 0 \\ 0, & \text{se } j = 1, \cdots, L-1 \\ f, & \text{se } j = L \end{cases} \quad \text{(VC2)}$$

$$\sum_{(k,k+l_i)\in A} z_{k,k+l_i} \geq d_i, i = 1, \cdots, m \quad \text{(VC3)}$$

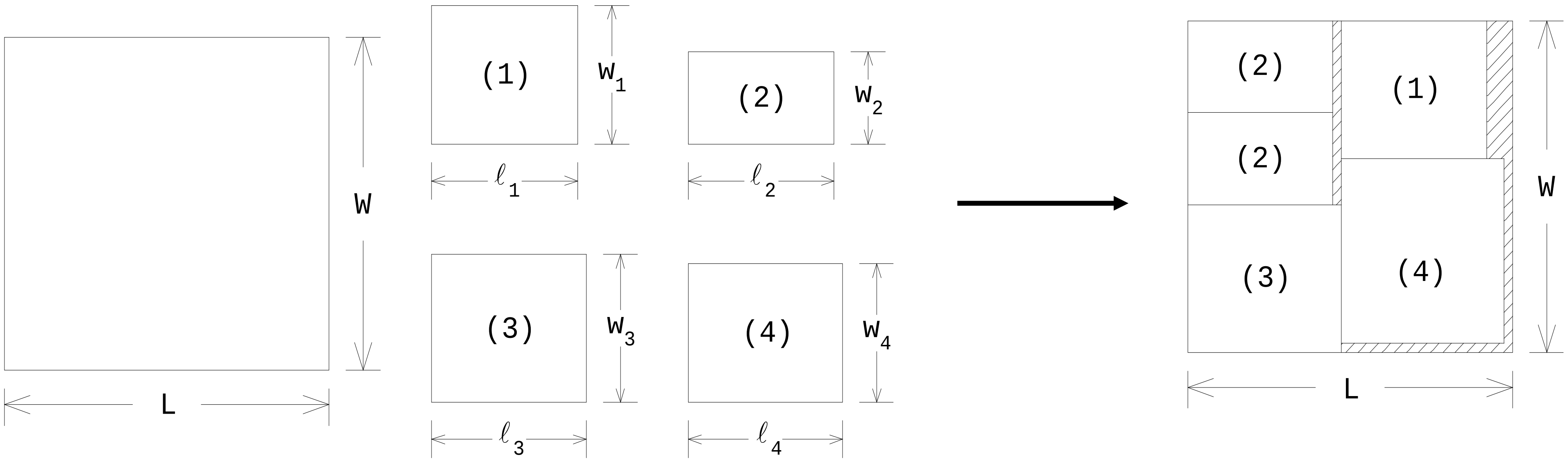
$$z_{ij} \in Z^+, \forall (i,j) \in A \quad \text{(VC4)}$$

# Problema de corte de estoque Bidimensional

# PCE Bidimensional



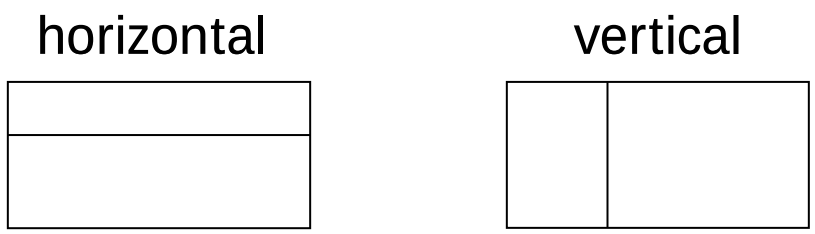
Objeto | Itens demandados | Padrão de corte



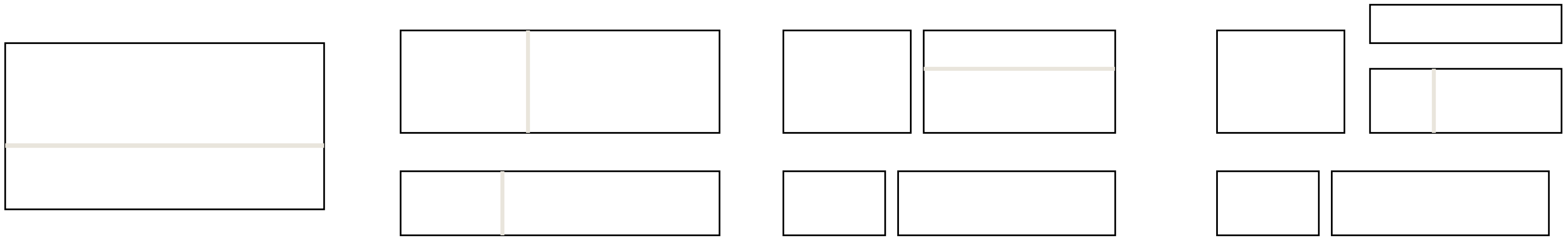

$$a=(1,\ 2,\ 1,\ 1)^t$$

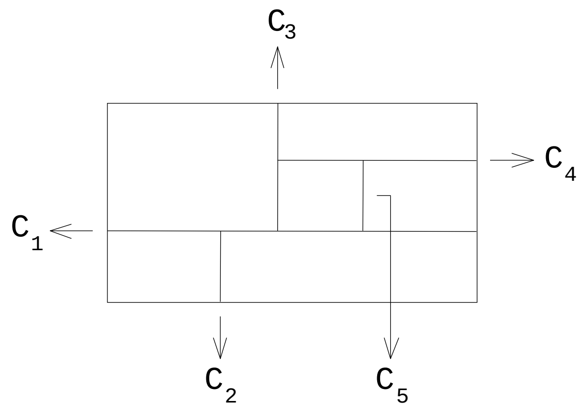
# PCE Bidimensional

<u>Corte guilhotinado:</u>

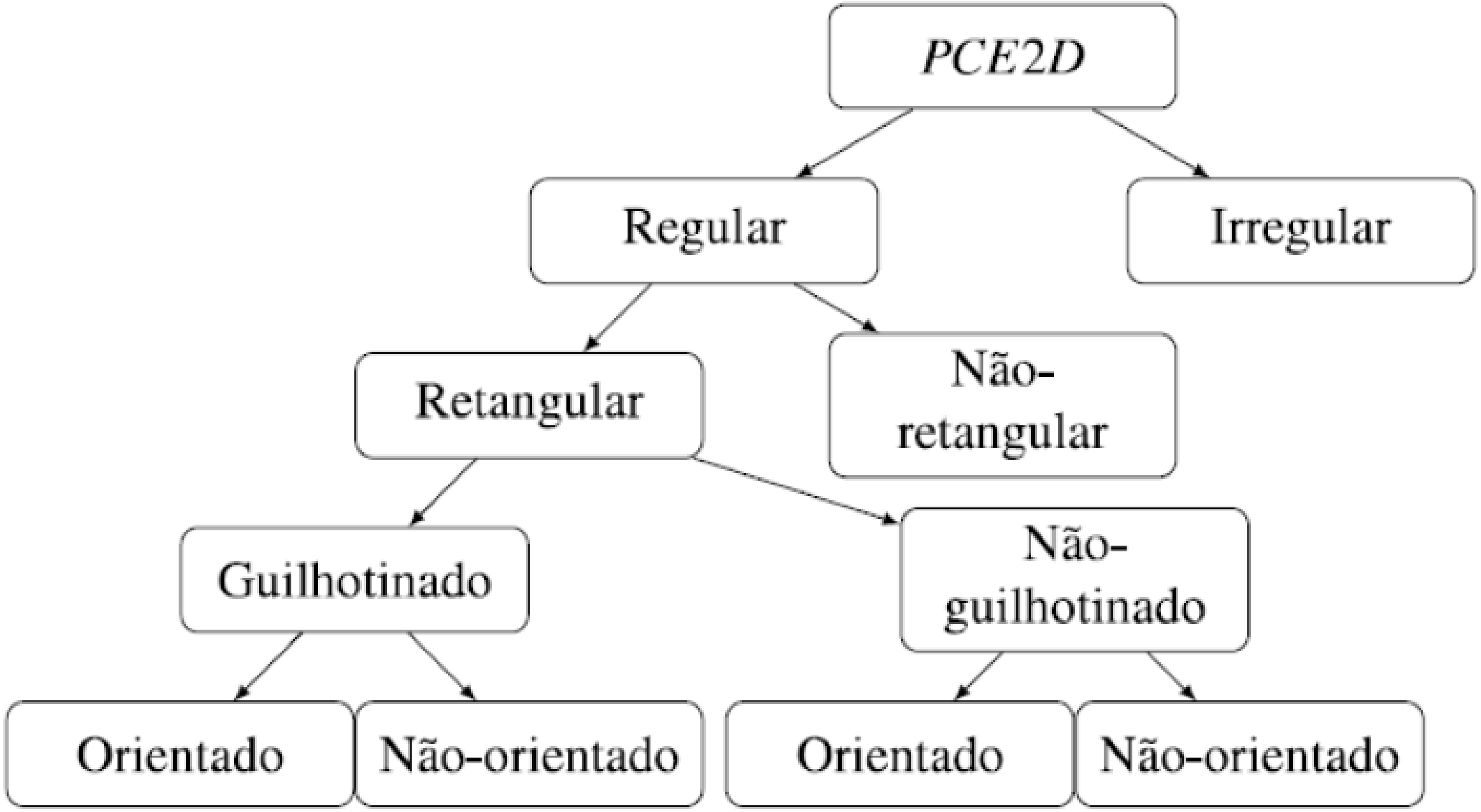



Padrão de corte guilhotinado:



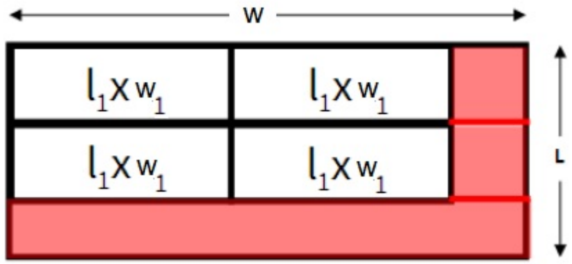


# PCE Bidimensional

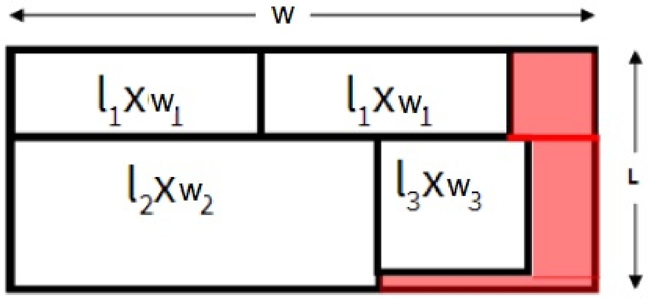



Fonte: Suliman 2006

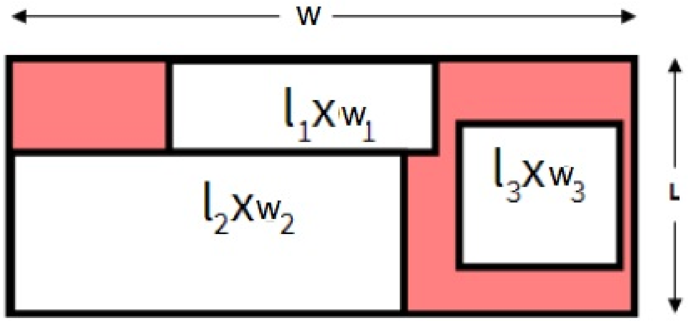
# PCE Bidimensional




- guilhotinado,
- homogêneo,
- 2-estágios, sem apara




- guilhotinado,
- não-homogêneo,
- 2-estágios, com apara




- Não guilhotinado

# PCE Bidimensional – Geração de Colunas

## Problema Mestre

$$\min \sum_{j=1}^{n} x_j$$

$$\text{sujeito a :} \begin{cases} \sum_{j=1}^{n} \boldsymbol{a}_j x_j \geq \boldsymbol{d} \\ x_j \in \mathbb{R}^+, \qquad \forall\ j \end{cases}$$

$\boldsymbol{\pi}$

$\boldsymbol{a}$

## Subproblema

$$\max \sum_{i=1}^{m} (l_i \pi_i) a_i$$

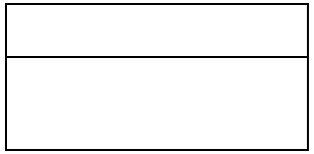
$$\text{sujeito a :} \begin{cases} \boldsymbol{a}\text{: } padrão\ bidimensional \\ a_i \in \mathbb{Z}^+, \qquad i =, 1, \dots, m \end{cases}$$

# PCE Bidimensional

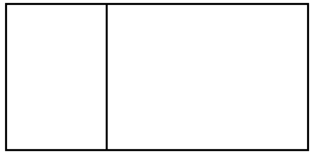
Corte guilhotinado

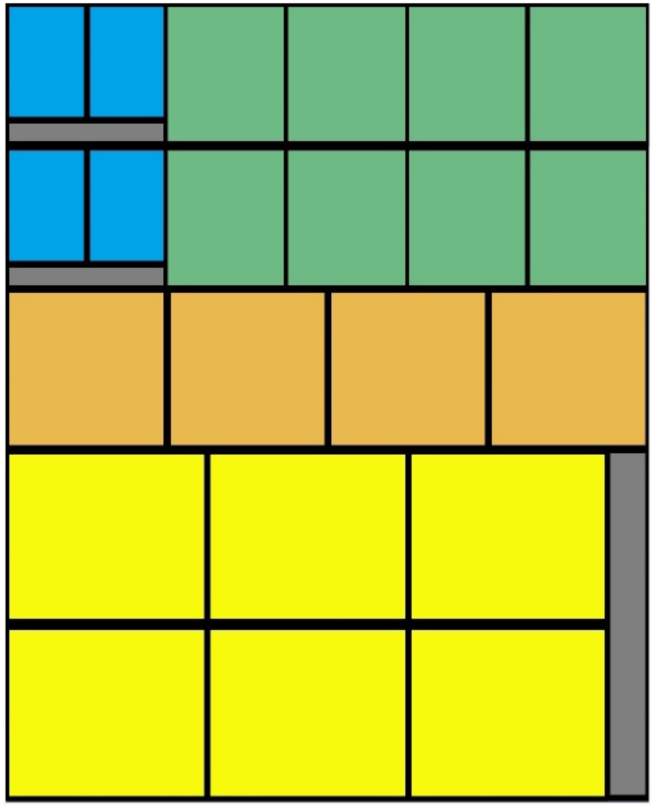
2 estágios (Gilmore e Gomory, 1965)

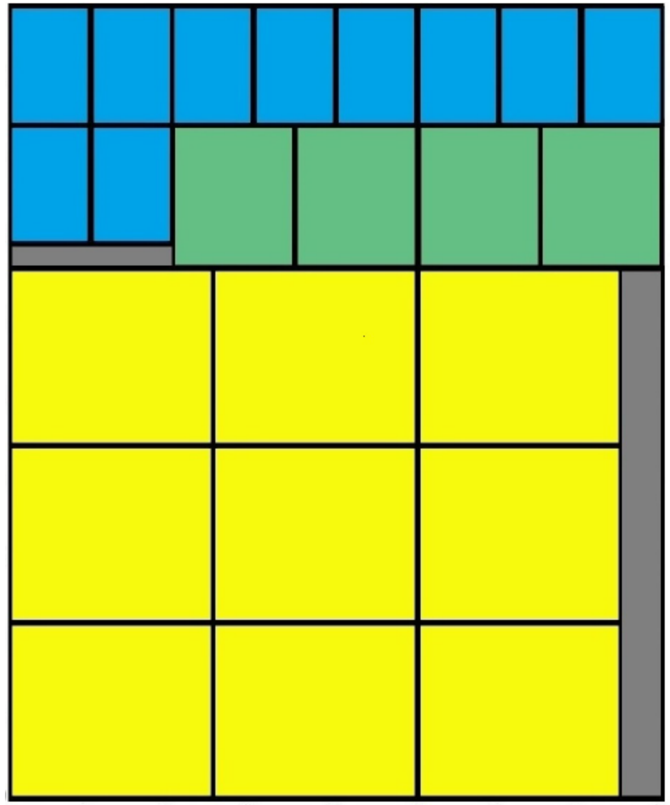
horizontal



vertical





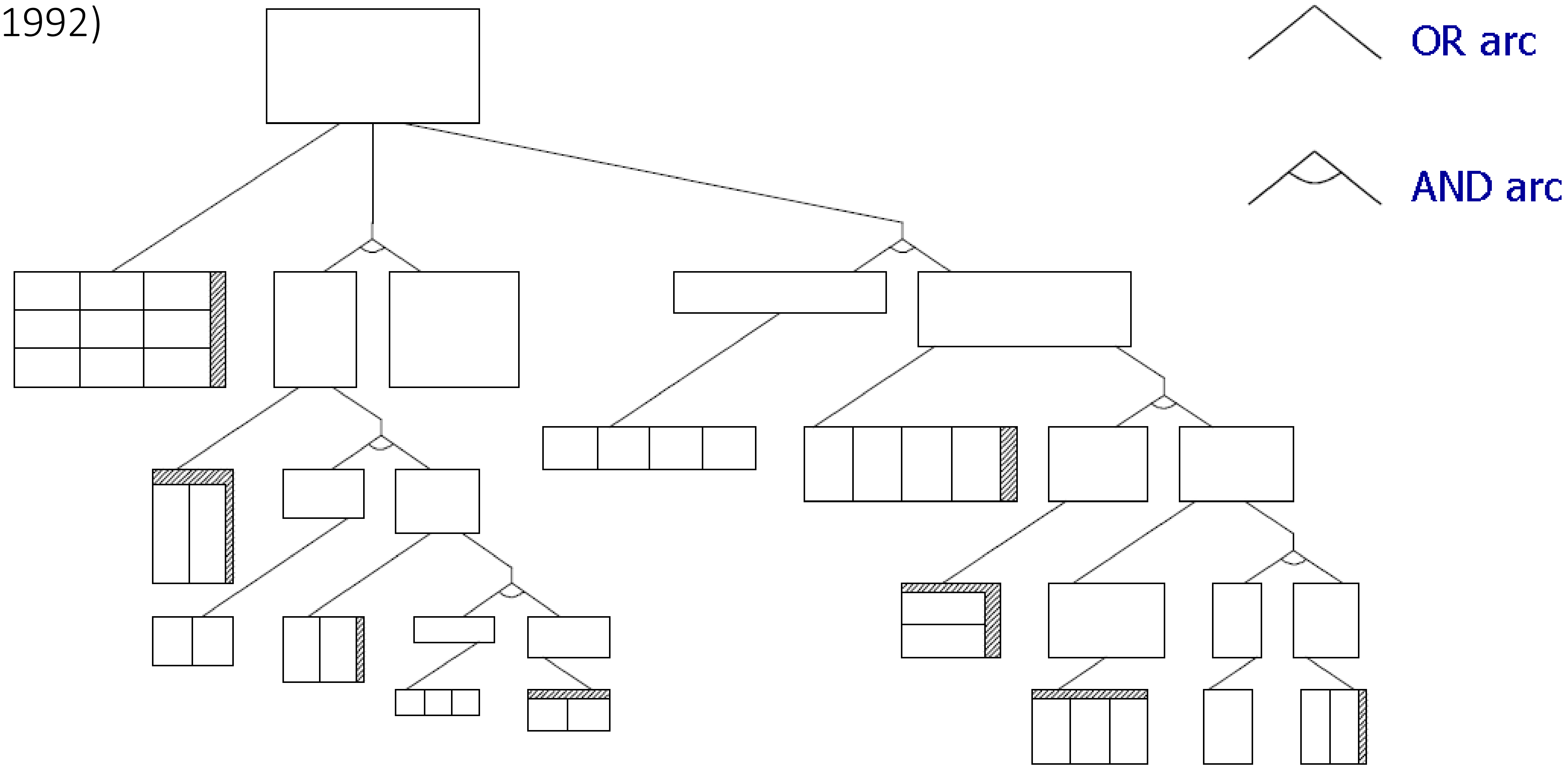


# PCE Bidimensional



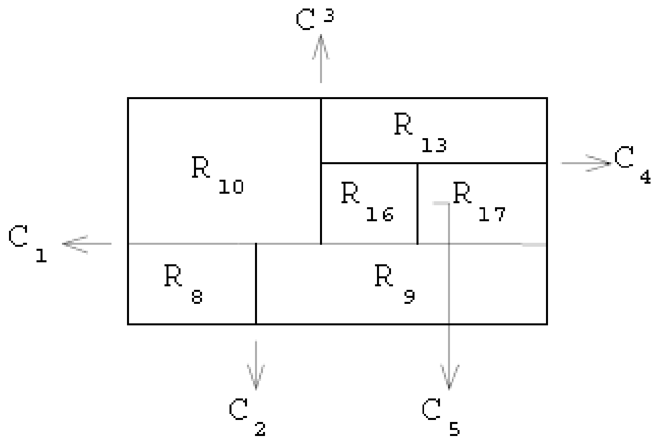
## Abordagem grafo e/ou

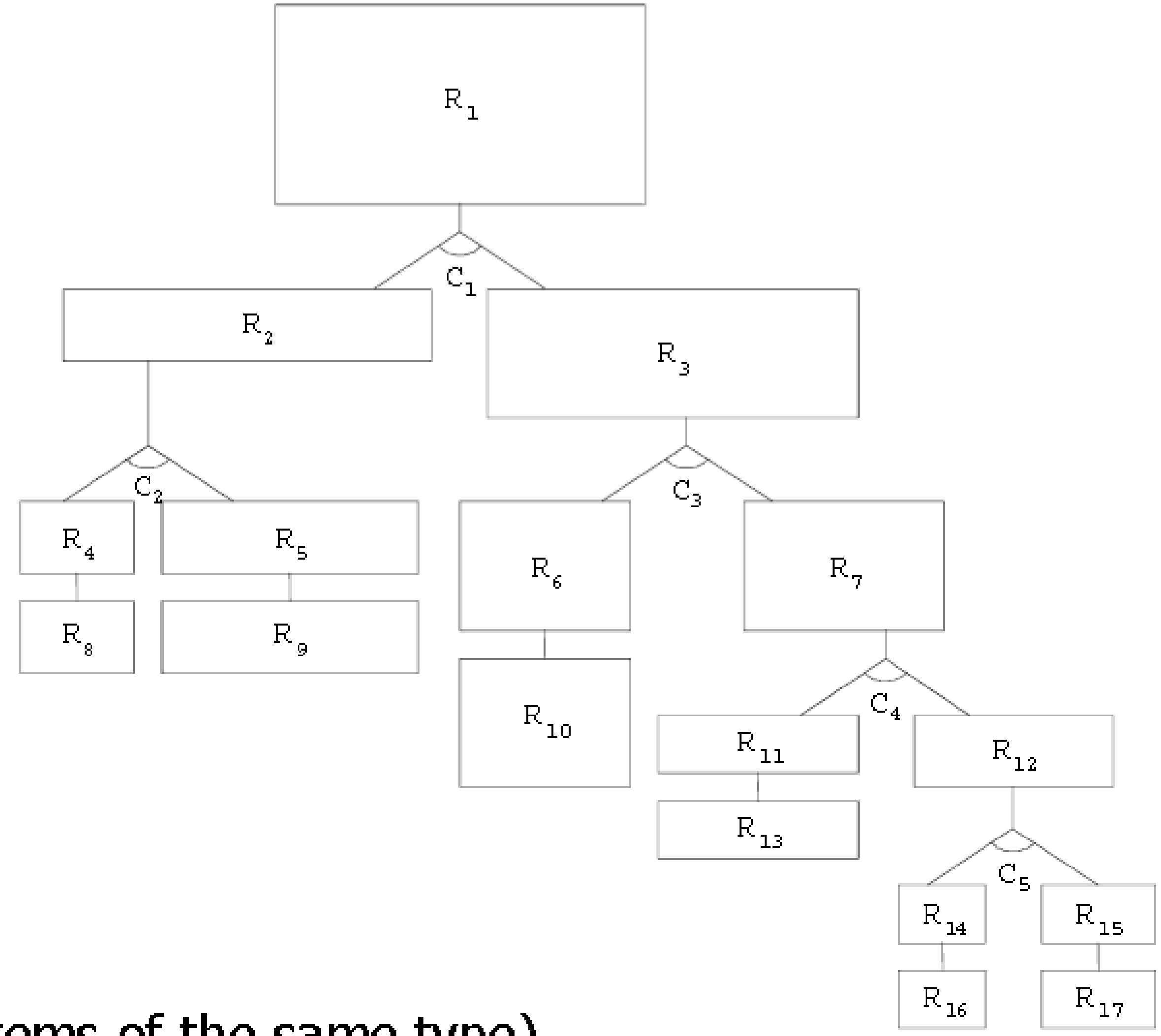
(Morabito, 1992)



## Abordagem grafo e/ou

(Morabito, 1992)




Initial node: plate (L,W)

Final node: 0-cut (one or more items of the same type)

(0-cut: symbolic cut to end the process)

LIMEIRA 2019
51º SBPO

# Obrigada!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FAPESP